.

Abstract

Let

$$E_x(q,a) := \sum_{\substack{n\leq x\\ n\equiv a(q)}} \Lambda(n) - \frac{x}{\phi(q)}.$$

A result of Hooley [1] says that for Q close to x (in the Barban-Davenport-Halberstam sense)

$$\sum_{q\leq Q} \sum_{a=1}^{q}{}' E_x(q,a)^3 \ll Q^2 \left(\frac{x}{Q}\right)^{3/2} e^{-c\sqrt{\log x/Q}}$$

and recently we proved that the error is in fact

$$Q^2 \left(\frac{x}{Q}\right)^{1+\epsilon}.$$

We discuss the relevance of the result and the main idea in the proof.

[1] C. Hooley - *On the Barban-Davenport-Halberstam theorem VIII* - Journal für die reine und angewandte Mathematik, 499 (1998)